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13. ABSTRACT (Maximum 200 words)

An analytic theory for the large (passive) scalar derivative skewness for turbulent transport in a mean scalar gradient has been devised using Lie Algebraic methods to solve the Hopf equation. This Reynolds independent skewness strongly contradicts Kolmogorov's theory occurs in shear flows, which are being studied numerically, emphasizing the analogies between scalar and momentum transport.

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I. Turbulent Transport of a Passive Scalar and Momentum

During this final, 4th year, no cost extension of my AFOSR grant, my research has focused on the fluctuations of a passive scalar in a linear externally imposed mean gradient and is evolving towards the same considerations for momentum i.e., the fluctuations in Reynolds stress in a boundary layer. The two problems are quantitatively similar.

A. Scalar Fluctuations

Within the last several years, we have refined our calculations for the pronounced exponential tails seen in the one point stationary scalar P.D.F. sustained by a linear gradient and acted upon by homogeneous turbulence. This effect was predicted by Pumir, Shraiman and myself and subsequently verified experimentally by Warhaft and Gollub with their respective collaborators.

Our current calculations have gone beyond the previous phenomenology and express via path integrals, the probability that the history averaged strain acting on a parcel of fluid is small. Such parcels can then be transported from further away along the mean gradient and thus give rise to tails in the scalar distribution, which we show are exponential. The history averaged strain rate is not a new concept and has been mentioned in various papers of John Lumley. Lacking was a means for its calculation, and path integrals naturally do the job. An essential ingredient to the observed exponential shape of the PDF's is the exponential divergence of near by trajectories. This property of non zero strain rate - or positive Lyapanov exponent is virtually a definition of turbulence and is not shared by pure shear i.e., $v_x = a(t) + b(t)y$ even if time dependent. Majda has analyzed this case in a number of papers (e.g., Phys. Fluids **A5** 1963, (1993)) and we consider this velocity field

to be a different problem.

The PDF of scalar differences, structure functions, or moments of the scalar gradient are logically problems that follow a study of the scalar itself. However some puzzling experiments on shear flows from the late 70's make these topics quite central to an understanding of turbulence. The experiments demonstrated a Reynolds' number independent derivative skewness or roughly linear 3rd order structure function. These are properties normally associated with the longitudinal velocity and any Kolmogorov like theory for the scalar predicts them to vanish strongly with Reynolds' number. This effect is not a particularity of shear flow as recent numerical and laboratory experiments at Cornell on homogeneous turbulence show. However the effect does disappear for a white noise velocity field.

We have found path integrals ill suited to handling a velocity field which scales and has a physically sensible correlation time. Instead we employ the Hopf equation for the three and four point correlation functions. With a plausible representation for non white velocity fields, the Hopf equation can be solved by Lie Algebraic methods, so the problem is integrable in a sense.

B. Shear Flows

There is no need to emphasize the relevance of turbulent boundary layers to all manner of engineering and defense related problems. The fluctuations well into the log layer resemble very much the slightly simpler problem of homogeneous shear as experimental investigations of Corrsin and simulations of Moin and others have shown. There is no wall in homogeneous shear, but merely a mean velocity $v_x = sy$ and an initial turbulence scale L which grows down stream in the experiments or in time in the simulations.

Alain Pumir during a recent visit to Cornell funded by the A.F.O.S.R. has simulated a homogeneous shear under stationary conditions with the computational box setting and fixing the integral scale of the turbulence. This had not been done before. Naturally one asks if the small scales, to the extent one can given the scale range, look Kolmogorov like. The velocity spectra are reasonable, but the skewness $Sk = < (\partial_y \tilde{v}_x)^3 > / < (\partial_y \tilde{v}_x)^2 >^{\frac{3}{2}}$ is order 1 and Reynolds independent. A similar result was found by Wallace experimentally in a boundary layer. The v_x involved is the fluctuating component, but with or without the mean velocity included, Kolmogorov theory predicts $Sk \sim s / < (\partial_y v_x)^2 >^{\frac{1}{2}} \sim Re^{\frac{1}{2}}$.

The nature of this violation of Kolmogorov theory is precisely analogous to the scalar derivative skewness and we believe has a similar origin phenomenologically. Mixing of scalar or v_x occurs efficiently in blobs – making the total v_x roughly y independent and expelling the imposed mean gradient into vortex sheets between the blobs. The blobs are of course transient. Pumir has also found numerically that the PDF of the Reynolds' stress conditioned upon v_x, v_y is very similar to the Nusseult number conditioned upon v_y and the scalar.

Therefore the quintessentially nonlinear problem of boundary layer turbulence has several features in common with scalar transport. Further quantitative comparisons should be possible when the theory for the scalar has advanced further. Given such close correspondence between the transport of scalar and momentum down a mean gradient there is hope for understanding how this similarity arises at the level of the governing equations.

Papers Published or Submitted

1. Lagrangian Path Integrals and Fluctuations in Random Flow, by B. Shraiman and E.D. Siggia, Phys. Rev. E **49**, 2912 (1994).
2. Turbulent Mixing of a Passive Scalar in Two Dimensions, by M. Holzer and E. D. Siggia, Phys. of Fluids **6**, 1820 (1994).
3. Simulations of Steady State Homogeneous Shear, by A. Pumir, (in preparation).

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2. A. Pumir senior visitors from Nice.